

# Projectile electron losses in the collisions with neutral targets: sudden-perturbation approximation

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## Abstract

An approach based on the sudden-perturbation approximation is presented for the treatment of single and multiple electron losses of high and intermediate energy projectiles in their collisions with neutral targets. Using this approach, we calculate electron loss cross sections of hydrogen-like Pb<sup>81+</sup> and Au<sup>78+</sup> projectiles in the collisions with neutral atoms. Also, multiple electron loss cross sections are calculated for U<sup>10+</sup> and U<sup>28+</sup> projectiles in their collisions with N<sub>2</sub> and Ar targets.

## 1. Introduction

High and low  $Z$  heavy projectiles having the electronic structure are of importance for various experimental studies at heavy-ion colliders [1–7]. In addition, heavy-ion beams are becoming an important tool in medicine and material technology. Therefore electron loss by intermediate and high-energy projectiles is of considerable interest both from fundamental and practical viewpoints.

It is expected that the forthcoming International FAIR facility provides a broad variety of atomic physics experiments with heavy ions including heavy-ion collisions with neutral targets [8]. The range of collision energies in this facility is expected to be from few MeV/amu to few GeV/amu. Such experiments need precise theoretical cross sections of the processes to be studied. An important application of relativistic highly charged ions with the electronic structure is the CERN neutrino beta-beam project [9], which is the subject of extensive discussion for past few years.

The theoretical treatment of the stripping process, which is often called electron loss, is complicated for the cases of multiple electron loss and intermediate and high energies due to the breaking of the perturbation theory. In these cases, the condition for applicability of the Born approximation,  $Z/v \ll 1$  (we use atomic units throughout the paper), where  $Z$  is

the charge of the projectile and  $v$  is the collision velocity, is often cannot be obeyed. Then one should use the so-called non-perturbative methods, which are applicable beyond the Born approximation [10, 11]. Breakdown of the perturbation theory has been confirmed in recent experiments, too [7].

Another important feature of (multiple) projectile electron loss in the collisions with neutral targets is the screening effect. The ion consists of a nucleus and the electronic structure, which partially screen the nuclear charge.

First treatment of the projectile K-shell ionization has been done within the framework of the Born approximation by Anholt *et al* [12] where a modified form-factor is used to account for target screening effect. More detailed treatment and numerical cross sections using the Born approximation for single electron loss from an arbitrary shell in the collisions with the neutral targets have been presented recently by Shevelko *et al* [13] who used LOSS computer code to account for the atomic structure of the target.

Strictly speaking, the collision process of such projectiles should be treated by considering excitations of both target and projectile electronic structures. The theory of simultaneous excitation and ionization of two colliding systems based on the Born approximation has been developed by several authors [14, 15] (see also [13]). Extension of these results to the case of relativistic collisions is done in [16, 17].

Most of the calculations of the projectile ionization cross sections have been performed using the classical trajectory Monte Carlo calculations [2, 3]. A quantum mechanical non-perturbative approach is also used recently [18]. The cases of relativistic [19] and ultra-relativistic [20] collisions are also considered. Recently, multiple electron losses in the collisions of low and high  $Z$  projectiles with neutral targets have attracted considerable attention both in theoretical and experimental contexts [1, 2]. Multiple (up to 15) loss cross sections have been measured in recent experiments [1, 2]. It should be noted that the non-perturbative quantum mechanical calculations of the multiple electron loss cross sections of high and low  $Z$  projectiles in their collisions with neutral targets have not yet been performed. In this paper, we use the sudden approximation to calculate such multiple ionization cross sections of  $U^{10+}$  and  $U^{28+}$  projectiles. This approximation was developed in [21] for the non-relativistic case, and its relativistic extension has been used for the treatment of target excitation and ionization in fast ion–atom collisions [22]. This paper is organized as follows. In the next section, we give a brief description of the sudden approximation. At the end of this section, we briefly describe relativistic extension of this approach. In section 3 we apply sudden-perturbation approach for the treatment of single ionization of relativistic projectiles. Section 4 presents a multiple ionization mechanism within the framework of sudden approximation. Finally, in section 5, we present the multiple electron loss cross sections for various collisions and their discussion.

## 2. Sudden approximation

We assume that both projectile and target move with the constant velocities along the linear trajectories. Also, for simplicity, we consider projectile and target as having only one electron each (then we will give extension to the multi-electron case). Then the projectile–target interaction potential is given by

$$V(\mathbf{r}_a, \mathbf{r}_p, t) = \frac{Z_p Z_a}{|\mathbf{R}_p - \mathbf{R}_a|} - \frac{Z_p}{|\mathbf{R}_p - \mathbf{R}_a - \mathbf{r}_a|} - \frac{Z_a}{|\mathbf{R}_p - \mathbf{R}_a + \mathbf{r}_p|} + \frac{1}{|\mathbf{R}_p - \mathbf{R}_a + \mathbf{r}_p - \mathbf{r}_a|},$$

where  $Z_p$  is the projectile charge,  $Z_a$  is the charge of the target nucleus, with  $\mathbf{R}_p = \mathbf{b}_p + \mathbf{v}_p t$  being the coordinate of the projectile nucleus,  $\mathbf{b}_p$  is the impact parameter with respect to the

origin,  $\mathbf{v}_p$  is the projectile velocity,  $\mathbf{R}_a = \mathbf{b}_a + \mathbf{v}_a t$  and  $\mathbf{r}_a$  are the target nucleus and electron coordinates, respectively.

The terms of this potential are defined as follows:

- $Z_p Z_a / |\mathbf{R}_p - \mathbf{R}_a|$  describes the interaction of the target and projectile nuclei;
- $-Z_p / |\mathbf{R}_p - \mathbf{R}_a - \mathbf{r}_a|$  is the interaction between the target electron and projectile nucleus;
- $-Z_a / |\mathbf{R}_p - \mathbf{R}_a + \mathbf{r}_p|$  is the interaction of the projectile electron with the target nucleus;
- $1 / |\mathbf{R}_p - \mathbf{R}_a + \mathbf{r}_p - \mathbf{r}_a|$  describes the interaction of the projectile electron with the target electron.

For the calculations of inelastic transitions in the electron states, we can write this potential as follows:

$$V(\mathbf{r}_a, \mathbf{r}_p, t) = -\frac{Z_p}{|\mathbf{R}(t) - \mathbf{r}_a|} - \frac{Z_a}{|\mathbf{R}(t) + \mathbf{r}_p|} + \frac{1}{|\mathbf{R}(t) + \mathbf{r}_p - \mathbf{r}_a|}, \quad (1)$$

with

$$\mathbf{R}(t) = \mathbf{R}_p - \mathbf{R}_a = \mathbf{b} + \mathbf{v}t$$

being the internuclear distance,  $\mathbf{b} = \mathbf{b}_p - \mathbf{b}_a$  the relative impact parameter, and  $\mathbf{v} = \mathbf{v}_p - \mathbf{v}_a$  is the relative velocity; the internuclear interaction is neglected as it does not contribute to the cross section.

Let  $\varphi_n(\mathbf{r}_a)$  and  $\psi_k(\mathbf{r}_p)$  be the target electron and projectile electron wavefunctions, respectively.

Then the initial states of the colliding systems are given by

$$\Phi_{00} = \psi_0(\mathbf{r}_p)\varphi_0(\mathbf{r}_a),$$

while the final state is

$$\Phi_{kn} = \psi_k(\mathbf{r}_p)\varphi_n(\mathbf{r}_a).$$

Our consideration of the fast ion–atom collisions is based on the so-called sudden approximation [23, 24], which is applicable for  $Z/v \ll 1$  (where  $Z$  is the projectile charge and  $v$  is the collision velocity). This approach is used recently to describe target excitation and ionization in fast highly charged ion–atom collisions [24].

It should be noted that the approach is closely related to the well-known Glauber approximation [23–26]. Also, in the ultra-relativistic limit, the sudden approximation reproduces the exact solution of the time-dependent Dirac equation, describing the ion–atom collision [27] (see also [28]).

Thus the sudden approximation is applicable when the Born approximation cannot be used provided the collision time is much less than the characteristic periods of the unperturbed system that allows one to solve the problem without restricting the value of perturbation. Usually, such a situation is possible when the collision velocity is rather high. Therefore the condition for applicability of the sudden approximation can be written as  $\tau_c \ll \tau_s$ , with  $\tau_c$  and  $\tau_s$  being the collision time and the characteristic period of the target or projectile electrons, respectively.

Thus evolution of the initial state in sudden approximation is given by [28]

$$\Phi_{00}(t) = \exp\left(-i \int_{-\infty}^t V(\mathbf{r}_a, \mathbf{r}_p, t) dt\right) \Phi_{00}. \quad (2)$$

Correspondingly, the evolution of the final state can be written as

$$\Phi_{kn}(t) = \exp\left(i \int_t^{+\infty} V(\mathbf{r}_a, \mathbf{r}_p, t) dt\right) \Phi_{kn}. \quad (3)$$

Then the transition amplitude for the target electron (from state  $\varphi_0(\mathbf{r}_a)$  to a state  $\varphi_n(\mathbf{r}_a)$ ) and for the projectile electron (from state  $\psi_0(\mathbf{r}_p)$  to a state  $\psi_k(\mathbf{r}_p)$ ) is described by the projection of state (2) to state (3):

$$A_{0 \rightarrow n}^{0 \rightarrow n} = \langle \Phi_{kn}(t) | \Phi_{00}(t) \rangle = \langle \Phi_{kn} | \exp \left( -i \int_{-\infty}^{+\infty} V(\mathbf{r}_a, \mathbf{r}_p, t) dt \right) | \Phi_{00} \rangle. \quad (4)$$

The corresponding probability is given by

$$w_{0 \rightarrow k}^{0 \rightarrow n} = |A_{0 \rightarrow k}^{0 \rightarrow n}|^2.$$

We will consider the collisions which lead to the definite transitions in both target and projectile electron states. Then summing over all the final states of the target electron and taking into account completeness condition

$$\sum_n \langle \varphi_n(\mathbf{r}'_a) | \varphi_n(\mathbf{r}_a) \rangle = \delta(\mathbf{r}'_a - \mathbf{r}_a),$$

we get

$$\sum_n w_{0 \rightarrow k}^{0 \rightarrow n} = \int d^3 \mathbf{r}_a |\varphi_0(\mathbf{r}_a)|^2 \left| \int d^3 \mathbf{r}_p \psi_k^*(\mathbf{r}_p) \exp \left( -i \int_{-\infty}^{+\infty} U_a(\mathbf{r}_a, \mathbf{r}_p, t) dt \right) \psi_0(\mathbf{r}_p) \right|^2, \quad (5)$$

where  $U_a(\mathbf{r}_a, \mathbf{r}_p, t)$  is the potential of interaction between the projectile electron and target:

$$U_a(\mathbf{r}_a, \mathbf{r}_p, t) = -\frac{Z_a}{|\mathbf{R}(t) + \mathbf{r}_p|} + \frac{1}{|\mathbf{R}(t) + \mathbf{r}_p - \mathbf{r}_a|}. \quad (6)$$

Thus based on the sudden approximation, we derived the transition amplitude for the projectile electron from state  $\psi_0$  to state  $\psi_k$ ,

$$W_{0 \rightarrow k}(\mathbf{b}) = \sum_n w_{0 \rightarrow k}^{0 \rightarrow n}, \quad (7)$$

as a function of the impact parameter  $\mathbf{b}$ .

Expression (5) is the averaged excitation probability of the projectile electron. The corresponding cross section can be obtained by integrating equation (7) over the impact parameter:

$$\sigma = \int d^2 \mathbf{b} W_{0 \rightarrow k}(\mathbf{b}). \quad (8)$$

Consider the collision of a projectile having  $N_p$  electrons with a target having  $N_a$  electrons. Then the potential describing the interaction of target and projectile electrons can be written as

$$U_a(\{\mathbf{r}_a\}, \{\mathbf{r}_p\}, t) = -Z_a \sum_{p=1}^{p=N_p} \frac{1}{|\mathbf{R}(t) + \mathbf{r}_p|} + \sum_{p=1, a=1}^{p=N_p, a=N_a} \frac{1}{|\mathbf{R}(t) + \mathbf{r}_p - \mathbf{r}_a|}. \quad (9)$$

As is seen from this expression,  $U_a(\{\mathbf{r}_a\}, \{\mathbf{r}_p\}, t)$  is the function of the internuclear distance,  $\mathbf{R} = (vt, \mathbf{b})$ , target electrons coordinates,  $\{\mathbf{r}_a\}$  and projectile electrons coordinates,  $\{\mathbf{r}_p\}$ . Therefore equation (5) can be extended to the case of a multi-electron target and projectile and written as

$$W_{0 \rightarrow n}(\mathbf{b}) = \langle \varphi_0(\{\mathbf{r}_a\}) | \langle \Psi_n(\{\mathbf{r}_p\}) | \exp \left( -i \int_{-\infty}^{+\infty} U_a(\{\mathbf{r}_a\}, \{\mathbf{r}_p\}, t) dt \right) | \Psi_0(\{\mathbf{r}_p\}) \rangle | \varphi_0(\{\mathbf{r}_a\}) \rangle. \quad (10)$$

with  $|\Psi_0(\{\mathbf{r}_p\})\rangle$  and  $|\Psi_n(\{\mathbf{r}_p\})\rangle$  being the ground state and an arbitrary state, respectively. It should be noted that in derivation of this expression we were not interested in transitions to a specific state, but we sum over all final states in the target states, suggesting that there may be any transitions.

A direct use of this formula is rather complicated, especially for the cases when both target and projectile are multi-electron systems, so that  $N_a \gg 1$  and  $N_p \gg 1$ . However, the following simplification is possible: during the collision time, target electrons do not change their positions and the potential of interaction of the target and projectile electrons can be represented as an average of the potential (9) over the initial states of the target. Furthermore, we assume that target electrons are described via Dirac–Hartree–Fock–Slater one-electron orbitals [29].

Then a simple analytical form of the screening function for the neutral targets with the atomic numbers  $Z_a = 1 - 92$  can be presented [29]. The electron density can be written as

$$\rho_a(\mathbf{r}) = \frac{Z_a}{4\pi|\mathbf{r}|} \sum_{i=1}^{i=3} A_i \alpha_i^2 e^{-\alpha_i|\mathbf{r}|}, \quad (11)$$

where  $A_i$  and  $\alpha_i$  are the constants [29] which can be extracted from proper tables. Therefore the potential of interaction between target and projectile electrons can be represented as (we assume that the projectile is fixed at the origin and consider the target as a moving system, which corresponds to the replacement  $\mathbf{R}(t) \rightarrow -\mathbf{R}(t)$ )

$$U_a(\{\mathbf{r}_a\}, \{\mathbf{r}_p\}, t) = - \sum_{p=1}^{p=N_p} \frac{Z_a}{|\mathbf{R}(t) - \mathbf{r}_p|} \sum_{i=1}^{i=3} A_i e^{-\alpha_i|\mathbf{R}(t) - \mathbf{r}_p|}. \quad (12)$$

Then

$$\int_{-\infty}^{+\infty} U_a(\{\mathbf{r}_a\}, \{\mathbf{r}_p\}, t) dt = \sum_{p=1}^{p=N_p} \chi(\mathbf{b}, \mathbf{r}_p), \quad (13)$$

where  $\chi(\mathbf{b}, \mathbf{r}_p)$  has the meaning of the eikonal phase and is given by

$$\chi(\mathbf{b}, \mathbf{r}_p) = -\frac{2Z_a}{v} \sum_{i=1}^{i=3} A_i K_0(\alpha_i|\mathbf{b} - \mathbf{s}_p|), \quad (14)$$

with  $\mathbf{s}_p$  being the projection of  $\mathbf{r}_p$  onto the impact parameter,  $\mathbf{b}$ , plane. Thus the potential  $U_a$  in equation (10) does not depend on the target electron coordinates,  $\{\mathbf{r}_a\}$ , and because of the condition  $\langle \varphi_0(\{\mathbf{r}_a\}) | \varphi_0(\{\mathbf{r}_a\}) \rangle = 1$ , the probability of transition of the projectile electron from the ground state,  $|\Psi_0(\{\mathbf{r}_p\})\rangle$ , to an arbitrary excited state  $|\Psi_n(\{\mathbf{r}_p\})\rangle$ , provided any transition can occur with the target electron (which is given by equation (10)), can be written as

$$W_{0 \rightarrow n}(\mathbf{b}) = |\langle \Psi_n(\{\mathbf{r}_p\}) | \exp \left( -i \sum_{p=1}^{p=N_p} \chi(\mathbf{b}, \mathbf{r}_p) \right) | \Psi_0(\{\mathbf{r}_p\}) \rangle|^2. \quad (15)$$

Equation (15) describes the excitation probability of a projectile having velocity  $-\mathbf{v}$  in its collisions with a neutral target which is described as an extended object with a spatially non-uniform charge density,  $-\rho_a(\mathbf{r})$  (corresponding to the charge of nucleus  $Z_a$ ) which is given by equation (11). Such an interpretation allows us to extend equation (15) to the case of relativistic collisions. Again, we consider the ion's position as fixed and the target as moving along the  $x$ -axis. Then in the eikonal approximation, the transition probability (from state

$|\Psi_0(\{\mathbf{r}_p\})\rangle$  with energy  $E_0$  to a state  $|\Psi_n(\{\mathbf{r}_p\})\rangle$  with energy  $E_n$  is given by [30, 31]

$$W_{0 \rightarrow n}(\mathbf{b}) = \left| \langle \Psi_n | \left[ 1 - \exp \left\{ -i \sum_{p=1}^{N_p} \chi(\mathbf{b}, \mathbf{r}_p) \right\} \right] \gamma^{-N_p} S^{-2} \right. \\ \left. \times \exp \left[ i \sum_{p=1}^{N_p} \frac{x_p v}{c^2} (E_n - E_0) \right] | \Psi_0 \rangle \right|^2 \quad (16)$$

where  $\gamma = 1/\sqrt{1 - v^2/c^2}$ ,  $c$  is the light speed,  $\mathbf{b}$  is the impact parameter,  $S^{-2} = \prod_{p=1}^{N_p} S_p^{-2}$ , with  $S_p$  being the Lorentz matrix acting on the projectile electron's wavefunction with the bispinor index  $p$  (corresponding to the Dirac matrix,  $\alpha_p$ ). This matrix is given by  $S_p^{-2} = \gamma(1 - \mathbf{v}\alpha_p/c)$ .

If the projectile electrons remain as non-relativistic before and after collision then in equation (16) the wavefunctions  $\Psi_0$  and  $\Psi_n$  can be considered as two-component spinors and one can write  $\gamma^{-N_p} S^{-2} = 1$  and  $\exp \left[ i \sum_p v x_p (E_n - E_0)/c^2 \right] = 1$ . Therefore under these assumptions equation (16) gives equation (15). Therefore equation (15) is applicable in the case of relativistic collisions, too, provided projectile electrons remain as non-relativistic before and after collision [30, 31]. In this case, the eikonal phase in equation (15) is the same for relativistic and non-relativistic collisions.

### 3. Single ionization of the relativistic projectiles

In this section, we apply the above prescription to calculate electron loss of a relativistic hydrogen-like projectile in the collision with a neutral atom.

To simplify our treatment, we will consider the target as moving while projectile's position as fixed. Then the cross section of transition from state  $|0\rangle$  to a state  $|n\rangle$  according to (8), (14) and (15) can be written as

$$\sigma_n = \int |\langle n | 1 - \exp\{-i\chi(|\mathbf{b} - \mathbf{s}_p|)\} | 0 \rangle|^2 d^2\mathbf{b}, \quad (17)$$

where

$$\chi(|\mathbf{b} - \mathbf{s}_p|) = -\frac{2Z_a}{v} \sum_{i=1}^3 A_i K_0(\alpha_i |\mathbf{b} - \mathbf{s}_p|). \quad (18)$$

Equations (17) and (18) need an additional comment. In equation (17) it is supposed that collision velocity is relativistic (in the ion's rest frame) and the projectile electron is considered as non-relativistic before and after collisions. The eikonal phase in equation (18) corresponds to the excitation of the hydrogen-like atom projectile in the field of the neutral target. Therefore (unlike the case of target ionization, treated in [30, 31]) the integral (over the impact parameter) in equation (17) is convergent for all values of the impact parameter and in the case of projectile ionization, after integration,  $\gamma$ -dependence in equation (18) disappears. This can be explained as follows. First of all, it should be noted that in the case of target ionization (excitation) (as in [30, 31]) when transitions in the target electron state occur under the influence of the projectile nucleus Coulomb field, the eikonal phase is the standard integral from the Coulomb potential and the integral over the impact parameter diverges at higher values of  $b$ . Therefore in this case the cutoff of the integral should be performed at some upper limit,  $b_0$  and for  $b > b_0$  one should use dipole approximation, the so-called Bethe asymptotics [30, 31]. Namely, the Bethe asymptotics gives the logarithmic dependence on  $\gamma$  in the cross section.

**Table 1.** Electron loss cross sections (in kb) for the  $\text{Pb}^{81+}$  in the collisions 33 TeV (at 158 GeV/ $n$ ) with neutral atoms. Third and fourth columns present two sets of experimental data from [4] for single projectile loss. In the last column, results by Anholt and Becker [12] are presented.

Target	$Z_a$	$\sigma_i$ [4]	$\sigma_i$ [4]	$\sigma_i$ , (our results)	$\sigma_i$ [12]
Ar	18	$1.88 \pm 0.13$	$1.97 \pm 0.14$	2.303	3.7
Kr	36	$6.80 \pm 0.48$	$7.38 \pm 0.52$	8.460	13.2
Xe	54	$15.5 \pm 1.1$	$15.7 \pm 1.1$	17.843	29.4

In the case of the projectile ionization/excitation, the Coulomb field of the target is strongly screened and the static electric field (equation (12)) of the (neutral) target exponentially decreases at large impact parameters. This leads to the convergence of the integral in equation (13) and there is no need to use the Bethe asymptotics and matching procedure used in the case of the target ionization/excitation as it was done in [30, 31], and therefore the cross does not contain the  $\gamma$ -dependence.

Using the Fourier transformation

$$1 - \exp\{-i\chi(|\mathbf{b} - \mathbf{s}_p|)\} = \int a(\mathbf{q}) \exp\{i\mathbf{q}\mathbf{s}_p\} \frac{d^2\mathbf{q}}{(2\pi)^2}. \quad (19)$$

where

$$a(\mathbf{q}) = \int (1 - \exp\{-i\chi(|\mathbf{b} - \mathbf{s}_p|)\}) \exp\{-i\mathbf{q}\mathbf{s}_p\} d^2\mathbf{s}_p. \quad (20)$$

The matrix element in equation (17) can be written as

$$I_n = \langle n | 1 - \exp\{-i\chi(|\mathbf{b} - \mathbf{s}_p|)\} | 0 \rangle = \int a(\mathbf{q}) \langle n | \exp\{i\mathbf{q}\mathbf{s}_p\} | 0 \rangle \frac{d^2\mathbf{q}}{(2\pi)^2}. \quad (21)$$

Equation (20) can be written as

$$a(\mathbf{q}) = \int (1 - \exp\{-i\chi(s)\}) \exp\{-i\mathbf{q}(\mathbf{s} + \mathbf{b})\} d^2\mathbf{s} = 2\pi \exp\{-i\mathbf{q}\mathbf{b}\} f(q), \quad (22)$$

where

$$f(q) = \int (1 - \exp\{-i\chi(s)\}) J_0(qs) s ds, \quad \chi(s) = -\frac{2Z_a}{v} \sum_{i=1}^3 A_i K_0(\alpha_i s). \quad (23)$$

Then the cross section can be written as

$$\sigma_n = \int |I_n|^2 d^2\mathbf{b} = \int a(\mathbf{q}) a^*(\mathbf{q}') \langle n | \exp\{i\mathbf{q}\mathbf{s}_p\} | 0 \rangle \langle 0 | \exp\{-i\mathbf{q}'\mathbf{s}_p\} | n \rangle \frac{d^2\mathbf{q} d^2\mathbf{q}'}{(2\pi)^4} d^2\mathbf{b}. \quad (24)$$

Inserting into this expression equation (22) and taking into account the integral representation of the delta-function, we have

$$\sigma_n = 2\pi \int_0^\infty |f(q)|^2 f_n(q) q dq, \quad (25)$$

where  $f_n(q) = |\langle n | \exp\{i\mathbf{q}\mathbf{s}_p\} | 0 \rangle|^2$ .

In table 1 we present electron loss cross sections for the hydrogen-like,  $\text{Pb}^{81+}$  projectile in the collision with Ar, Kr and Xe targets and compare them with the experimental data. As is seen from this table, theoretical cross sections obtained using non-perturbative approach are quite close to experimental ones.

**Table 2.** Electron loss cross sections (in barns) for Au<sup>78+</sup> projectile in the collisions with neutral atoms with the nucleus charge ( $Z_a = 6, 13, 29, 47, 70$ ) at 10, 8 GeV/ $n$ .

$Z_a$	Cross section [32] $\sigma_i$	Cross section (our results) $\sigma_i$	Cross section [12], $\sigma_i$
6	310.0 (30)	306.172	310.0
13	1180.0 (90)	1314.88	1280.0
29	5260.0 (500)	6172.58	5800.0
47	16200.0 (1400)	15040.2	14400.0
79	38200.0 (3200)	37467.2	38800.0

In table 2 the results for electron loss cross sections by the hydrogen-like Au<sup>78+</sup> projectile are presented and compared with the experimental [32] and other theoretical results, obtained within the framework of the Born approximation [12].

As it follows from table 1 our results agree with the experimental data rather well, though, strictly speaking, the electrons of the Pb<sup>81+</sup> and Au<sup>78+</sup> projectiles in the ground state should be considered as quasi-relativistic.

As is seen from this comparison, for higher values of  $Z_a$  the results of non-perturbative calculations are closer to experimental ones than the perturbation theory results. However, for low  $Z_a = 6, 13, 29$  the cross sections obtained within the Born approximation are close to the experimental data than the results of non-perturbative calculations. This can be explained by the fact that for those values  $Z_a$  perturbation theory is still valid.

#### 4. Multiple projectile electron losses

In order to use equation (15) for the description of multiple electron losses by the projectile, we use an approach which was used recently for the treatment of multiple ionizations of complex atoms in the collisions with fast highly charged ions [24, 30, 31, 33, 34]. According to this scheme, we will assume that the projectile electrons remain non-relativistic before and after collision and each electron is described by the hydrogen-like wavefunction. Then the initial and final states are given by the wavefunctions  $\Psi_0(\mathbf{r}_1, \dots, \mathbf{r}_{N_p}) = \prod_{i=1}^{N_p} \phi_i(\mathbf{r}_i)$ ,  $\Psi_f(\mathbf{r}_1, \dots, \mathbf{r}_{N_p}) = \prod_{i=1}^{N_p} \psi_i(\mathbf{r}_i)$ , respectively.

The total  $(N_p - N)$ -fold probability ionization of the  $N_p$ -electron projectile, which corresponds to the transition of  $N_p - N$  electrons into the continuum, and the transition of other  $N$  electrons into an arbitrary state of the discrete spectrum (with account of the unitarity) can be written as (15)

$$W^{(N_p-N)+}(\mathbf{b}) = \frac{N_p!}{(N_p - N)!N!} \prod_{i=1}^{N_p-N} p_i(\mathbf{b}) \prod_{j=N_p-N+1}^{N_p} (1 - p_j(\mathbf{b})), \quad (26)$$

$\prod_{j=N_p-N+1}^{N_p} (\dots) = 1$ , for  $N = 0$ , where

$$p_i(b) = \int d^3k_i \left| \int d^3r_i \psi_{\mathbf{k}_i}^*(\mathbf{r}_i) \exp\{-i\chi_i(\mathbf{b}, \mathbf{r}_i)\} \phi_i(\mathbf{r}_i) \right|^2 \quad (27)$$

is the generalized inelastic form-factor and  $\mathbf{k}_i$  is the momentum of the  $i$ th electron in the continuum.

Therefore introducing the average over the orbital momentum,  $l$ , and its projection,  $m$ , the inelastic one-electron form-factor can be written as

$$p(b) = \frac{1}{n_0} \sum_{n=1}^{n_0} \frac{1}{M_n} \sum_{l,m} \int d^3k \left| \int d^3r \psi_{\mathbf{k}}^*(\mathbf{r}) \exp\{-i\chi(\mathbf{b}, \mathbf{r})\} \phi_{nlm}(\mathbf{r}) \right|^2, \quad (28)$$

where summing is performed over the all possible values of  $l$  and  $m$  for a given  $n$ th shell,  $M_n$  is the number of such values,  $n$  is the principal quantum number,  $n_0$  is the number of shells. It is clear that for a large enough number of (projectile) electrons one can assume that  $p(b) = p(|\mathbf{b}|)$  does not depend on the angles of the vector  $\mathbf{b}$  and  $p(b)$  has the meaning of the averaged one-electron ionization probability in which the averaging is performed as follows: suppose that the projectile is in a state which has some distribution of electrons over the shells. Then we average over all one-electron states. After that averaging is performed over all filling methods. This gives the averaged one-electron loss probability,  $p(b)$ .

Then replacing in equation (26) each one-electron form-factor with the averaged one, we obtain the  $N_p - N$ -loss probability as [22, 35]

$$W^{(N_p - N)^+}(b) = \frac{N_p!}{(N_p - N)!N!} p(b)^{(N_p - N)} (1 - p(b))^N. \quad (29)$$

However, the effective projectile charge,  $Z^*$ , which is the charge ‘felt’ by the target, depends on the ionization multiplicity. To take this into account, we make the following substitutions in equation (28)  $\mathbf{k} = \mathbf{k}/Z^*$ ,  $\mathbf{b} = \mathbf{b}Z^*$ ,  $\mathbf{r} = \mathbf{r}Z^*$  that correspond to using Coulomb units [26].

Then the right-hand side of equation (28) can be calculated using the hydrogen wavefunctions instead of hydrogen-like functions, and the  $Z^*$ -dependence will be taken into account using the substitution  $b = bZ^*$ . Such a replacement allows us to calculate the cross section for more general form than that in the case of independent electron approximation [36]).

Consider first the total projectile ionization cross section, i.e. the ionization of all  $N_p$  electron. Then in equation (26)  $N = 0$  and  $W$  can be represented as the product of  $N_p$  one-electron form-factors. Introducing the effective charge corresponding to the total ionization of the projectile,  $Z_{N_p}^*$ , and replacing each form-factor with the average one from equation (28), we obtain the probability of the total ionization  $W^{N_p^+} = [p(b)]^{N_p}$ , where  $b \equiv bZ_{N_p}^*$ . In the case of the ionization of  $N_p - N$  electrons we have

$$W^{(N_p - N)^+}(\mathbf{b}) = \frac{N_p!}{(N_p - N)!N!} \sum_{m=0}^N (-1)^m \frac{N!}{(N - m)!m!} \{p(b)\}^{N_p - N + m}. \quad (30)$$

To obtain the corresponding cross section, we should insert equation (30) into equation (8) and integrate over the whole impact parameter plane. For integrating this, we need to know the explicit form of the function  $p(b)$ , which is defined by equation (28). It is difficult to calculate this function when the number of projectile electrons is large enough. However, some simplification is possible in the case of ionization of high multiplicity, i.e. for  $N_p \gg 1$ ,  $N_p - N \gg 1$  [24, 30, 31].

The integral (8) can be estimated (over  $d^2b$ ) using the Laplace method assuming that the function  $p(b)$  has one maximum located inside or on the (left) boundary of the integration interval,  $b = 0$ . The existence of such a maximum follows from the fact that the probability is positively defined and tends to zero for large impact parameters. First, we represent the probability as

$$[p(b)]^N = \exp[N \ln(p(b))] = \exp[Nf(b)].$$

Then for  $N \gg 1$  we have [37]

$$\int_{b_0}^{b_1} e^{-Nf(b)} g(b) db \sim \frac{G}{\mu} \Gamma\left(\frac{\lambda}{\mu}\right) e^{-Nf(b_0)} \left[\frac{1}{FN}\right]^{\lambda/\mu}, \quad (31)$$

where  $\Gamma(x)$  is the Gamma-function.  $G, \mu, \lambda, F$  are the numbers determined from the behaviour of the functions  $f(b)$  and  $g(b)$  near the maximum,  $b_0$ :

$$f(b) - f(b_0) \sim F(b - b_0)^\mu, \quad g(b) \sim G(b - b_0)^{\lambda-1}.$$

For the total cross section of  $N_p$ -electron loss, we have

$$\sigma^{N_p+} = 2\pi \frac{G}{(Z_{N_p}^*)^2} \frac{1}{\mu} \Gamma\left(\frac{\lambda}{\mu}\right) \left[\frac{1}{FN_p}\right]^{\lambda/\mu} [p(b_0)]^{N_p}. \quad (32)$$

In the case of  $(N_p - 1)$ -electron loss the probability is the difference of two different terms, one which contains the product of  $N_p - 1$  one-electron form-factors and corresponds to the  $N_p - 1$  electrons in the continuum while the second term contains the product of  $N_p$  one-electron form-factors. The effective charges corresponding to first and second terms are  $Z_{N_p-1}^*$  and  $Z_{N_p}^*$ , respectively. Integrating each term using the Laplace method, we obtain  $(N_p - 1)$ -electron loss cross section:

$$\sigma^{(N_p-1)+} = N_p \sigma^{N_p+} \left[ \left( \frac{Z_{N_0}^*}{Z_{N_p-1}^*} \right)^2 \left( \frac{N_p}{N_p - 1} \right)^{\lambda/\mu} \frac{1}{p(b_0)} - 1 \right]. \quad (33)$$

Similarly, in the case of  $(N_p - N)$ -electron loss we get

$$\begin{aligned} \sigma^{(N_p-N)+} &= \frac{N_p! \sigma^{N_p+}}{(N_p - N)! N!} \sum_{m=0}^N (-1)^m \left( \frac{Z_{N_p}^*}{Z_{N_p-N+m}^*} \right)^2 \frac{N!}{(N - m)! m!} \\ &\times \left( \frac{N_p}{(N_p - N + m)} \right)^{\lambda/\mu} \{p(b_0)\}^{-N+m}, \end{aligned} \quad (34)$$

where  $Z_{N_p-N+m}^*$  is the effective charge for  $(N_p - N + m)$ -electron loss.

Equations (32), (33) and (34) allow us to calculate projectile electron loss of any multiplicity (provided  $N_p \gg 1$ ,  $(N_p - N) \gg 1$ ), using some two known cross sections.

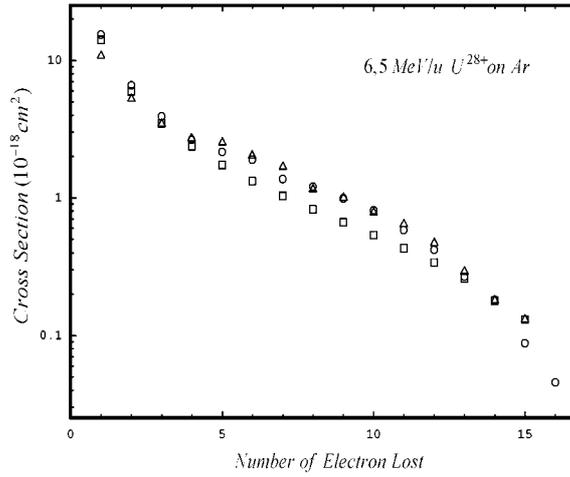
If the cross sections  $\sigma^{N_p+}$  and  $\sigma^{(N_p-1)+}$  are known one can easily find  $p(b_0)$  as

$$\frac{1}{p(b_0)} = \left( \frac{Z_{N_0-1}^*}{Z_{N_p}^*} \right)^2 \left( \frac{N_p - 1}{N_p} \right)^{\lambda/\mu} \left( \frac{\sigma^{(N_p-1)+}}{N_p \sigma^{N_p+}} + 1 \right). \quad (35)$$

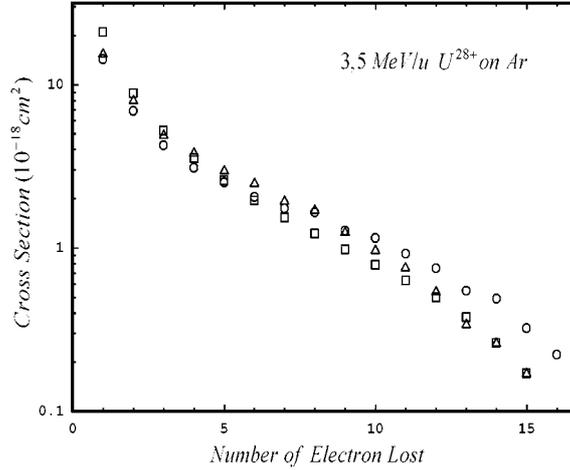
Inserting this into equation (34) the  $(N_p - N)$ -electron loss cross section can be expressed in terms of the known cross sections  $\sigma^{N_p+}$  and  $\sigma^{(N_p-1)+}$ . As these known cross sections one can use either experimental values or (if available) theoretical ones which satisfy the condition of applicability of equations (32)–(34),  $N_p - N \gg 1$ .

## 5. Results and discussion

In figures 1–5 the multiple electron loss cross sections of the uranium projectile in the collisions with argon and nitrogen targets calculated using the above scheme are presented and compared with the experimental data from [1, 2]. In these calculations, for the simplicity of all cases, we chose  $N_p$  as equal to  $N_m$ —the maximal number of lost electrons for which experimental data are available [1, 2]. In these calculations, effective charge was taken as  $Z_N^* = Q + N$ , where  $Q$  is the projectile charge before collision.



**Figure 1.** Multiple electron loss cross section (in  $10^{-18} \text{ cm}^2$ ) in the collision of  $6.5 \text{ MeV/u } U^{28+}$  ion with Ar. The triangles are the experimental data [2], the circles are the results of Monte Carlo calculations from [2], and the squares are the results of our calculations.

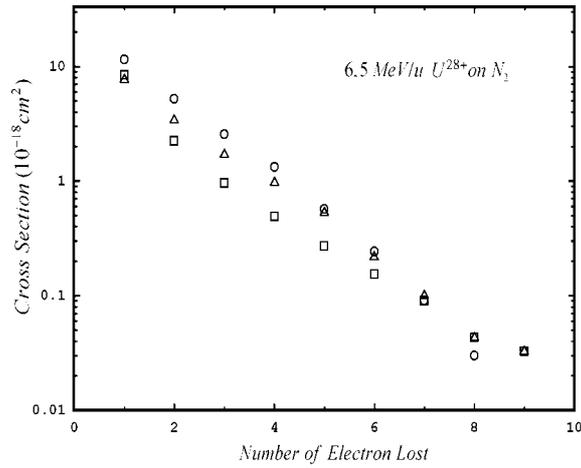


**Figure 2.** Dependence of the multiple electron loss cross section (in  $10^{-18} \text{ cm}^2$ ) on the number of lost electrons in the collision of the  $3.5 \text{ MeV/u } U^{28+}$  ion with Ar. The triangles are the experimental data [2], the circles are the results of Monte Carlo calculations from [2], and the squares are the results of our calculations.

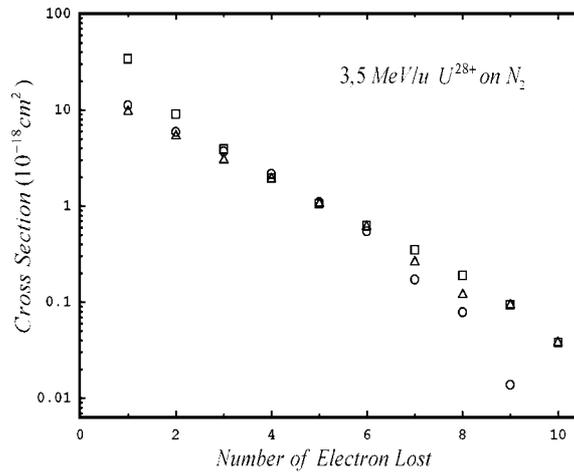
The values of  $\lambda/\mu$  and  $N_p$  were chosen as follows:  $\lambda/\mu = 1.5$  for  $U^{28+} + \text{Ar}$ ,  $N_p = 15$  (for figures 1 and 2);  $\lambda/\mu = 2.5$  for  $U^{28+} + N_2$ ,  $N_p = 9$  (for figure 3) and  $N_p = 10$  (for figure 4);  $\lambda/\mu = 1.5$  for  $U^{10+} + \text{Ar}$ ,  $N_p = 13$  (for figure 5).

As the known cross sections,  $\sigma^{N_p+}$  and  $\sigma^{(N_p-1)+}$ , we used experimental values from [2]. In all cases, the quantity  $\lambda/\mu$  is considered as a selectable parameter which causes  $N_p$ -dependence of this parameter for various targets and projectiles.

As is seen from the figures, the obtained results are in good correspondence with the experimental data even for losses of low multiplicity, i.e., out of the interval where equations (32) and (35) are valid.

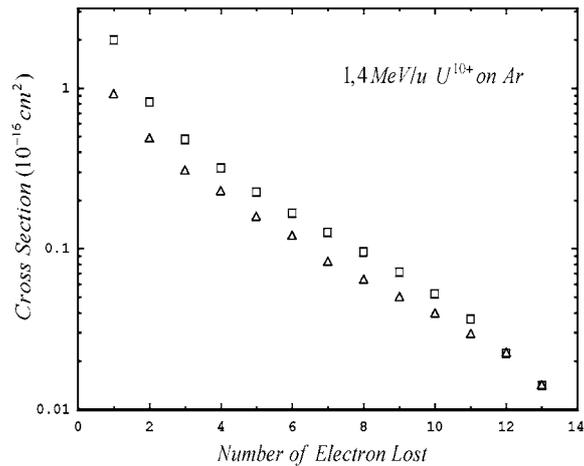


**Figure 3.** Dependence of the multiple electron loss cross section (in  $10^{-18} \text{ cm}^2$ ) on the number of lost electrons in the collision of the  $6.5 \text{ MeV}/u \text{ U}^{28+}$  ion with  $\text{N}_2$ . The triangles are the experimental data [2], the circles are the results of Monte Carlo calculations from [2], and the squares are the results of our calculations.



**Figure 4.** Dependence of the multiple electron loss cross section (in  $10^{-18} \text{ cm}^2$ ) on the number of lost electrons in the collision of the  $3.5 \text{ MeV}/u \text{ U}^{28+}$  ion with  $\text{N}_2$ . The triangles are the experimental data [2], the circles are the results of Monte Carlo calculations from [2], and the squares are the results of our calculations.

Thus in this work we have developed an approach based on sudden-perturbation approximation which allows one to treat projectile ionization and excitation beyond the Born approximation. The approach is applied to calculate single electron loss of relativistic projectiles in the collision of neutral atoms. Also, using this approach we obtained the recurrence formula using which the  $(N_p - N)$ -electron loss cross section for the  $N_p$ -electron projectile is expressed in terms of the  $N_p$ - and  $(N_p - 1)$ -electron loss cross sections. Using the above formalism, the cross section for the projectile stripping of any multiplicity can be calculated provided two cross sections are known. It should be noted that the choice of the known cross sections can be arbitrary and defined by the condition of applicability in



**Figure 5.** Dependence of the multiple electron loss cross section (in  $10^{-16} \text{ cm}^2$ ) on the number of lost electrons in the collision of the  $1.4 \text{ MeV/u } U^{10+}$  ion with Ar. Triangles are the experimental data [2], and squares are the results of our calculations.

equations (32)–(34),  $N_p - N \gg 1$ . The obtained results are applied to the calculations of multiple electron losses by  $U^{10+}$  and  $U^{28+}$  projectiles in their collisions with the neutral targets. Finally, we note that a more strict treatment should be performed by taking into account the so-called anti-screening effects which are described in [13].

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### References

- [1] DuBois R D *et al* 2004 *Phys. Rev. A* **70** 032712
- [2] Olson R E *et al* 2004 *J. Phys. B: At. Mol. Opt. Phys.* **37** 4539
- [3] Watson R L *et al* 2003 *Phys. Rev. A* **67** 022706
- [4] Krause H F *et al* 2001 *Phys. Rev. A* **63** 032711
- [5] Ludziejewski T *et al* 2000 *Phys. Rev. A* **61** 052706
- [6] Wells E *et al* 1999 *Phys. Rev. A* **60** 3734
- [7] Rymuza P *et al* 1993 *J. Phys. B: At. Mol. Opt. Phys.* **26** L169
- [8] GSI 2005 GSI Report 2005-1, June 2005
- [9] Zucchelli P 2002 *Phys. Lett. B* **532** 166 (<http://beta-beam.web.cern.ch/beta-beam/>)
- [10] Eichler J and Meyerhof W E 1995 *Relativistic Atomic Collisions* (San Diego: Academic)
- [11] Eichler J 1990 *Phys. Rep.* **193** 165
- [12] Anholt R *et al* 1985 *Phys. Rev. A* **32** 3302  
Anholt R and Becker U 1987 *Phys. Rev. A* **36** 4628
- [13] Shevelko V P, Tolstikhina I Yu and Stöhlker Th 2001 *Nucl. Instrum. Methods Phys. Res. B* **184** 295
- [14] Bates D R 1962 *Atomic and Molecular Processes* (Nova Torque: Academic)
- [15] Shevelko V P, Böhne D, Franzke B and Stokler Th 1999 *Atomic Physics with Heavy Ions* ed H F Beyer and V P Shevelko (Berlin: Springer) p 305

- [16] Voitkiv A B, Grün N and Scheid W 2000 *Phys. Rev. A* **61** 052704
- [17] Voitkiv A B and Najjari B 2004 *J. Phys. B: At. Mol. Opt. Phys.* **37** 3339
- [18] Voitkiv A B, Grün N and Scheid W 2000 *J. Phys. B: At. Mol. Opt. Phys.* **33** 3431
- [19] Matveev V I and Gusarevich E S 2003 *J. Exp. Theor. Phys.* **96** 34
- [20] Voitkiv A B, Müller C and Grün N 2000 *Phys. Rev. A* **62** 062701
- [21] Dykhne A M and Yudin D L 1978 *Sov. Phys.—Usp.* **21** 549
- [22] Matveev V I and Musakhanov M M 1994 *J. Exp. Theor. Phys.* **78** 149
- [23] Eichler J 1977 *Phys. Rev. A* **15** 1856
- [24] Matveev V I 1995 *Phys. Part. Nucl.* **26** 329
- [25] Matveev V I and Tolmanov S G 1995 *J. Exp. Theor. Phys.* **80** 989
- [26] Landau L D and Lifshitz E M 1989 *Quantum Mechanics: Non-Relativistic Theory* 3rd edn, vol 3 (New York: Pergamon)
- [27] Baltz A J 1997 *Phys. Rev. Lett.* **78** 1231
- [28] Matveev V I 2005 *Theor. Math. Phys.* **142** 48
- [29] Salvat F, Martinez J D, Mayol R and Parellada J 1987 *Phys. Rev. A* **36** 467
- [30] Matveev V I and Rakhimov Kh Yu 1998 *J. Exp. Theor. Phys.* **87** 891
- [31] Matveev V I, Rakhimov Kh Yu and Matrasulov D U 1999 *J. Phys. B: At. Mol. Opt. Phys.* **32** 3849
- [32] Claytor N, Belkacem A and Dinneen T *et al* 1997 *Phys. Rev. A* **55** R842
- [33] Berg H *et al* 1988 *J. Phys. B: At. Mol. Opt. Phys.* **21** 3929
- [34] Ullrich J *et al* 1989 *Nucl. Instrum. Methods Phys. Res. B* **40/41** 149
- [35] McGuire J H 1992 *Adv. At. Mol. Opt. Phys.* **29** 217
- [36] McGuire J H 1977 *Phys. Rev. A* **16** 41
- [37] Olver F W J 1974 *Introduction to Asymptotics and Special Functions* (New York: Academic)